Quadratic Model Exercise

Your homework is a project to implement a simple quadratic model in MATLAB.

When a basketball is thrown, its trajectory is a parabola, which is the shape associated with a quadratic model. Typically the motion of the ball has components in both the horizontal and vertical directions.

For this project we’re going to simplify the problem by modeling a ball that is tossed straight up into the air and comes straight back down, so it has vertical motion only. The height of the ball is well approximated as a quadratic function of time.

Before you get started on the project, let’s revisit the discussion questions from the lesson write-up. Here are some thoughts that may help you understand the model.

1. **What are the two forces acting on the objects in the horizontal and vertical directions?**

   There are two major forces acting on the objects that are thrown or launched. Before the object leaves the hand or launcher, the throwing force gives it an initial velocity in the horizontal direction or the vertical direction or both, depending on the situation. After the object is airborne, the only force acting on it is gravity, which acts in the vertical direction only.

2. **Are the forces dependent or independent?**

   The force of gravity is independent of the initial throwing force, so motion in the horizontal and vertical directions are independent.

3. **If you were to graph the path of the objects, what geometric object would that path match?**

   The motion of objects under the influence of gravity is approximated by a quadratic model. You probably noticed in the video clips that the trajectories looked like parabolas.

Here’s an explanation of the model we’ll be using.

You want to model the height of the ball in meters as a function of time. The model is a quadratic function:

$\boldsymbol{h(t)=gt^2+v_1t+h_1}$, where $\boldsymbol{t}$ is time, $\boldsymbol{h(t)}$ is height as a function of time, $\boldsymbol{g}=-9.81$ m/sec^2 is the gravitational acceleration, $\boldsymbol{v_1}$ is the initial velocity (velocity at time t=1, a constant), and $\boldsymbol{h_1}$ is the initial height (height at t=1, a constant). Notice that $\boldsymbol{g}$ is negative because gravity pulls the ball down.

The initial velocity $\boldsymbol{v_1}$ and the initial height $\boldsymbol{h_1}$ are constants that we assume are known.

We'll implement this model in MATLAB using a method called difference equations. We want to calculate $\boldsymbol{h(t)}$ over a period of time starting at an initial time $\boldsymbol{t_1}$ and ending at a final time $\boldsymbol{t_f}$. We'll calculate $\boldsymbol{h(t)}$ at a sequence of N+1 evenly spaced values of t in that range, including the end points.

The spacing of the time values is called the time step is designated $\boldsymbol{dt}$. At time $\boldsymbol{t_1}$, we know the height is $\boldsymbol{h_1}$ and the velocity is $\boldsymbol{v_1}$. I'm referring to them symbolically here, but they are numbers that you will know at the start of the simulation. If you know the height and velocity of the ball at time $\boldsymbol{t_1}$, and you know the forces acting on the ball, you can calculate the approximate height and velocity of the ball at time $\boldsymbol{t_2}$. They would be $\boldsymbol{h_2}$ and $\boldsymbol{v_2}$. Remember that $\boldsymbol{t_2}= \boldsymbol{t_1}+ \boldsymbol{dt}$. Once you know $\boldsymbol{h_2}$ and $\boldsymbol{v2}$, you can calculate $\boldsymbol{h_3}$ and $\boldsymbol{v_3}$, and so on.

At each time step you will calculate $\boldsymbol{h_{t+1}}$, the height at the next time step, by calculating the *change in height* and adding it to $\boldsymbol{h_t}$, the height at the current time step. The *change in height* is equal to $\boldsymbol{\frac{1}{2}\, g\, dt^2 + v_t\, dt}$. You must also update the velocity at each time step by calculating the *change in velocity* and adding it to $\boldsymbol{v_t}$, the current velocity to get $\boldsymbol{v_{t+1}}$. The *change in velocity* is equal $\boldsymbol{g\, dt}$, which is the acceleration due to gravity multiplied by the *change in time*.

In summary,

change in height = $\boldsymbol{\frac{1}{2}\, g\, dt^2 + v_t\, dt}$

$\boldsymbol{h_{t+1}} = \boldsymbol{h_t}$ + *change in height*

change in velocity = $\boldsymbol{g\, dt}$

$\boldsymbol{v_{t+1}} = \boldsymbol{v_t}$ + *change in velocity*

Do this for each time step. Remember that $\boldsymbol{h_1}$ and $\boldsymbol{v_1}$ are known at the start.