MODULE 4.1

Modeling Falling and Skydiving

Downloads

The text's website has available for download for various system dynamics tools the following files containing the models in this module: *Fall*, *FallFriction*, and *Fall-Skydive*.

Introduction

What is it like to skydive? Imagine ascending in a small plane to, say, 10,000 feet, when the jumpmaster opens the door. The jumpmaster asks you if you are ready to jump. You head for the door and walk out onto a step under the wing, holding on to a strut. You experience lots of wind and noise. Your heart is pounding wildly. The jumpmaster yells, "Go!" You arch your body and release your grip on the strut. Your adrenalin levels have never been higher as you plunge toward earth at 120 mph. Nevertheless, you are in control. For the next 50 seconds, simple body movements can alter your speed, direction and position. At three thousand feet, the landscape is fast approaching, and you pull your cord. As it deploys, your descent slows, and the mad rush of wind ceases, replaced by the rustling sounds of your canopy. Soon you gently settle to the ground.

The use of parachutes or parachute-like devices to slow the descent of jumpers from positions of considerable height may have begun with the twelfth-century Chinese. However, the first evidence of a parachute in the western world appeared in the late-fifteenth-century drawings of Leonardo da Vinci. His pyramid-shaped design was to be constructed of linen and a wooden frame. There is no record of Leonardo experimenting with his invention, but late last century it was demonstrated successfully.

Not much development of parachutes took place until late in the eighteenth century, when hot-air balloons were being shown across Europe. Andres-Jacques Garnerin, a French balloonist of dubious reputation, was one of the first persons to demonstrate a parachute without a rigid frame. He successfully descended from his balloon (which exploded) at about 3000 feet using a gondola suspended by an umbrella-shaped parachute.

Jumps using parachutes from airplanes began in the early twentieth century but were primarily for rescuing observation balloon pilots. Barnstormers performed parachute-jumping demonstrations at air shows during the time between the world wars. During World War II, both sides exploited the capabilities of parachutes for dispersing men and supplies.

Sport parachuting (skydiving) probably has its roots in the first free-fall conducted in 1914, but the sport really gained popularity only in the 1950s and 1960s (Bates; Cislunar 1997).

In this module, using a system dynamics tool we model the motion of someone skydiving. Such a jump has two phases, a free-fall stage followed by a parachute stage with greater air friction. In preparation for development of this model, we reconsider the main example in Module 2.3 on "Rate of Change" and a follow-up exercise (Exercise 1) in Module 2.4 on "Fundamental Concepts of Integral Calculus" involving the motion of a ball thrown straight up from a bridge. We model this motion, first ignoring air friction and then refining the model to consider this additional force.

Acceleration, Velocity, and Position

The above-mentioned modules on calculus discuss how the instantaneous rate of change, or derivative, of position (s) with respect to time (t) is velocity (v), and the instantaneous rate of change of velocity with respect to time is acceleration (a). In derivative notation, we have the following:

$$v(t) = \frac{ds}{dt}$$

$$a(t) = \frac{dv}{dt}$$

In Example 1, we use these derivatives in modeling the main illustration from the module on "Rate of Change" (Module 2.3).

Quick Review Question 1

This question reflects on Step 2 of the modeling process—formulating a model—for developing a model for a falling object. We simplify this first attempt at a model by ignoring friction. After completing this question and before continuing in the text, we suggest that you develop a model for a falling object.

a. Determine four variables for the model and their units in the metric system.
b. Give a differential equation relating time (t), position (s), and velocity (v).
c. Give a differential equation relating time (t), velocity (v), and acceleration (a).
d. Ignoring friction, give any of the following that is constant in a fall: time, distance, velocity, acceleration. In a model diagram, we will store such a value in a converter/variable.

e. In a model diagram, list the components that will be in stocks (box variables): t, s, v, a, ds/dt, dv/dt.
f. In a model diagram, give the value(s) that will flow into the position stock (box variable) for change in position: t, s, v, a, ds/dt, dv/dt.
g. In a model diagram, give the value(s) that will flow into the velocity stock (box variable) for change in velocity: t, s, v, a, ds/dt, dv/dt.

Example 1

To model with a system dynamics tool the motion of a ball that someone throws straight up from a bridge, we have stocks for the quantities that accumulate, the height (*position*) and velocity (*velocity*) of the ball. During the simulation, we can observe their changing values in a graph and table. A flow representing the change goes into velocity (*change_in_velocity*). Change in velocity is acceleration, and in this case, the acceleration is due to gravity. Therefore, a converter/variable (*acceleration_due_to_gravity*) contains the constant for **acceleration due to gravity**, which with up being the positive direction is approximately **−9.81 m/sec²**. The converter connects to *change_in_velocity*, which has this constant as its equation. Also, the flow for the change in height (*change_in_position*) is identical to the current velocity, *velocity*. Thus, we have a connector from *velocity* to *change_in_position*, and define the value of this flow to be *velocity*. Because velocity can be positive, zero, or negative, we specify that the flow can go into or out of *position*. For flexibility in models that we derive from this one, we also make *change_in_velocity* a biflow. Moreover, we specify that *velocity* and *position* can take on negative as well as positive values. To match the example in the earlier modules, we initialize *velocity* to be 15 m/sec and *position* to be 11 m, which is the height of the bridge. Figure 4.1.1 presents a diagram for a model of motion of the ball with a white arrowhead on each flow, indicating the secondary biflow direction.

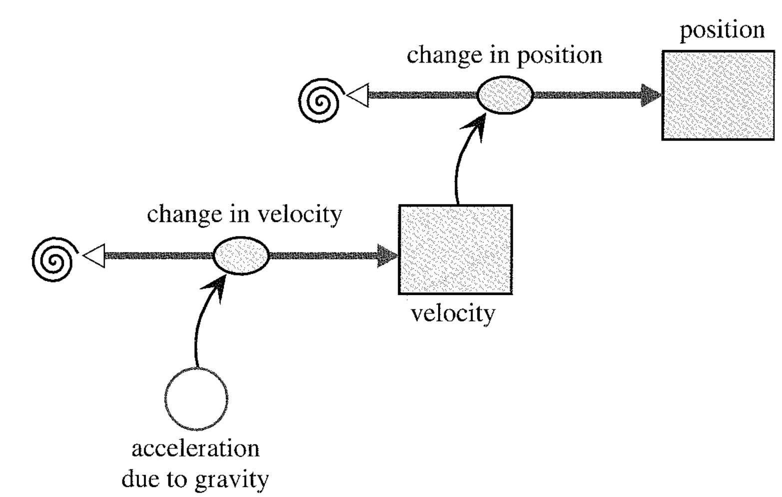

Figure 4.1.1 Diagram of motion of ball thrown straight up

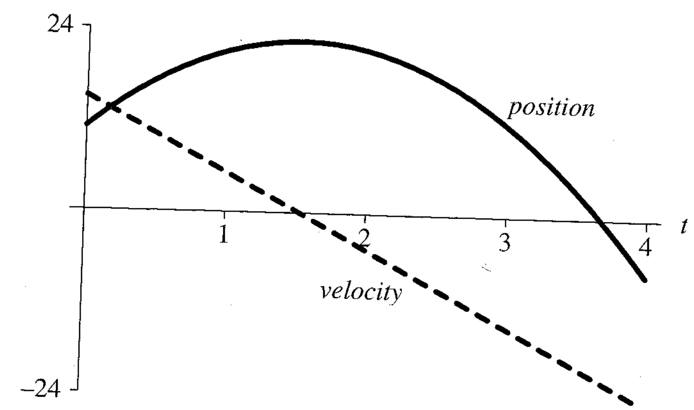

Figure 4.1.2 Graph of velocity (m/sec) and position (m) of ball versus time (sec)

Quick Review Question 2

Give the formula in metric units for each of the following components in Figure 4.1.1:

a. The converter *acceleration_due_to_gravity*
b. The flow *change_in_velocity*
c. The flow *change_in_position*

Output consists of a graph and a table of velocity and height versus time. With $\Delta t = 0.25$ sec and the Runge-Kutta 4 integration technique, which Module 5.4 discusses, we obtain a table of values that matches Table 2.3.1 in Module 2.3 on "Rate of Change." The graph of velocity in Figure 4.1.1 agrees with a similar graph (Figure 2.3.2) in that module. As Figure 4.1.2 shows, the graph of velocity versus time is the line $v(t) = 15 - 9.8t$.

For some of the models, it is more convenient to consider speed than velocity. The **speed** gives the magnitude of the change in position with respect to time, while

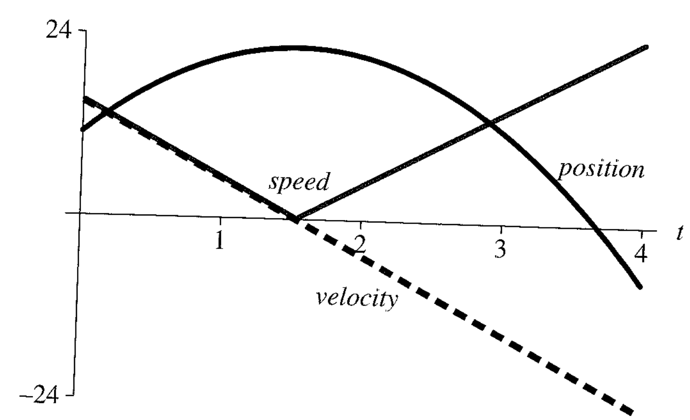

Figure 4.1.3 Graph of velocity (m/sec), position (m), and speed (m/sec) of ball versus time (sec)

the velocity expresses the magnitude with the direction. Thus, speed is the absolute value of the velocity. To incorporate speed, we have a connector/arrow from the *velocity* stock to a new converter/variable, *speed*, which stores the equation for the absolute value of velocity. The graph in Figure 4.1.3 shows speed and velocity decreasing in a linear fashion to 0 m/sec at about time 1.5 sec. Afterward, speed steadily increases.

Physics Background

Before developing additional examples of falling and skydiving, we need to consider some formulas from physics—Newton's Second Law and approximations of friction. Newton's Second Law concerns force applied to a mass imparting acceleration. So that we can refine models to account for air friction, we also consider several approximations of such a force.

Newton's Second Law has far-reaching significance. In this text, we employ the law in modeling situations from the motion of skydivers to the motion of the planets. The law states that a force F acting on a body of mass m gives the body acceleration a. Moreover, as the following models indicate, the acceleration is directly proportional to the force and inversely proportional to the mass:

$$a = F/m$$

or

$$F = ma$$

Newton's Second Law A force F acting on a body of mass m gives the body acceleration a according to the following formula:

$$\boldsymbol{F = ma}$$

We can apply this formula to obtain the relationship between weight and mass. **Weight** is a force and is not the same as mass. The acceleration involved is acceleration due to gravity, which is about -9.81 m/sec^2 or -32 ft/sec^2 for up being the positive direction. For example, an object that has mass of 20 kilograms (kg) has a weight of -196.2 newtons, as the following shows:

$$\text{weight} = F = (20\text{ kg})(-9.81\text{ m/sec}^2) = -196.2\text{ kg m/sec}^2 = -196.2\text{ newtons}$$

The metric unit for force is a **newton (N)** or kg m/sec^2.

Definition A **newton (N)** is a measure of force, and **1 N = 1 kg m/sec^2**.

Quick Review Question 3

Determine the following including units:

a. The mass of an object that weighs 981 N.
b. The acceleration that results when a net force of 10 N is applied to an object with mass 5 kg.

Kinetic friction or **drag**, too, is a force. This force between objects is in the opposite direction to a moving object and tends to slow motion. Thus, kinetic friction dampens motion of an object. When an object moves through a fluid, such as air or water, the fluid friction is a function of the object's velocity. For example, the faster we pedal a bicycle, the harder it is for us to do so. As our velocity increases, so does the friction of the air on our bodies.

Several models that estimate friction exist. In Module 8.3 on "Empirical Models," we study how to derive our own model, such as a model for drag, from data. In this module, we consider two models for drag on a body traveling through a fluid.

For a small object traveling slowly, such as a dust particle floating through the air, we usually employ **Stokes' friction**, which states that friction on the particle is approximately proportional to its velocity,

$$F = kv$$

where k in kg/sec is a constant of proportionality for the particular object and fluid, and v in m/sec is the velocity.

For a larger object moving faster through a fluid, we usually employ **Newtonian friction**, which states that the drag is approximately as follows:

$$F = 0.5CDAv^2$$

where C is a dimensionless constant of proportionality (the **coefficient of drag** or **drag coefficient**) related to the shape of the object, D is the density of the fluid, and A is the object's projected area in the direction of movement. For a particular situation, C, D, and A are constants, so that the drag is approximately proportional to the velocity squared. **At 0°C, the density of air at sea level is 1.29 kg/m³.** For shapes that are hydrodynamically good, $C < 1$; for spheres, C is about 1; and for shapes that are hydrodynamically inefficient, $C > 1$. Many objects have a coefficient of drag of about 1. Thus, through air with $C = 1$, Newtonian friction is approximately the following:

$$F = 0.65Av^2$$

The **density of water at 3.98°C**, where the fluid achieves its maximum density, is **1.00000 g/cm³**, yielding a formula with a different coefficient. Table 4.1.1 summarizes the three models for fluid friction considered here.

The drag force is in the opposite direction of motion, and the sign of velocity indicates the direction. On the upward portion of a trajectory, drag and gravity both act downward; while on the downward part, drag is upward, and gravity downward. Thus, for the general formula for Newtonian friction, we take the absolute value of only one of the velocity terms and multiply the entire formula by −1, yielding

Table 4.1.1
Summary of Several Models for Magnitude of Fluid Friction

Name	*Formula*	*Meanings of Symbols*	*When to Use*
Stokes' friction	$F = kv$	k constant v velocity	Very small object moving slowly through fluid
Newtonian friction	$F = 0.5CDAv^2$	C coefficient of drag D density of fluid A object's projected area in direction of movement v velocity	Larger objects moving faster through fluid
Newtonian friction through air	$F = 0.65Av^2$	A object's projected area in direction of movement v velocity	Larger objects with $C = 1$ moving faster through sea-level air

$-0.5CDAv|v|$. If *ABS* is the absolute value function, the translation of this formula into a system dynamics tool is as follows:

```
-0.5 * drag_coefficient * density * projected_area * velocity
    * ABS(velocity)
```

Quick Review Question 4

Calculate the following:

a. The density of 3.98°C water in kg/m³
b. The magnitude of friction in newtons of a ball falling through 3.98°C water, where the coefficient of drag is 0.9, the cross-sectional area of the ball is 0.03 m², and its velocity is −20 m/sec
c. Write the formula for Newtonian friction for a system dynamics tool, where the coefficient of drag is 1 and the air density is 1.29 kg/m³, namely, $-0.65Av|v|$, A and v are appropriate variables, and *ABS* is the absolute value function.

Quick Review Question 5

This question reflects on refinement of the model of an object falling through sea-level air to account for friction. After completing this question and before continuing in the text, we suggest that you revise the model in Example 1 to account for drag friction for practice in model development.

a. Give the inputs to compute drag friction.
b. Give a formula for air friction in a system dynamics tool's model with v for velocity, A for projected area, and *ABS* for the absolute value function.
c. Give the force(s) acting on the object.
d. Give a formula for an object's weight in a system dynamics tool's model, where g is the acceleration due to gravity and m is the mass of the object.

e. Give a formula for an object's acceleration in a system dynamics tool's model, where F is the total force on the object (weight + air friction) and m is the mass of an object.

Friction During Fall

Example 2

Example 1 to model the motion of a ball thrown straight up does not account for air friction. To do so, we consider two forces on the ball, gravity and drag friction. The force due to gravity is its weight, which by Newton's Second Law is $F = ma$. Thus, adjusting the model diagram in Figure 4.1.1, we include a converter/variable for *weight* with connections from converters/variables for *mass* and *acceleration_due_to_gravity* (see Figure 4.1.4). Newtonian friction for the air friction including direction is $F = -0.65Av|v|$. In the diagram, connectors/arrows go from *velocity* and

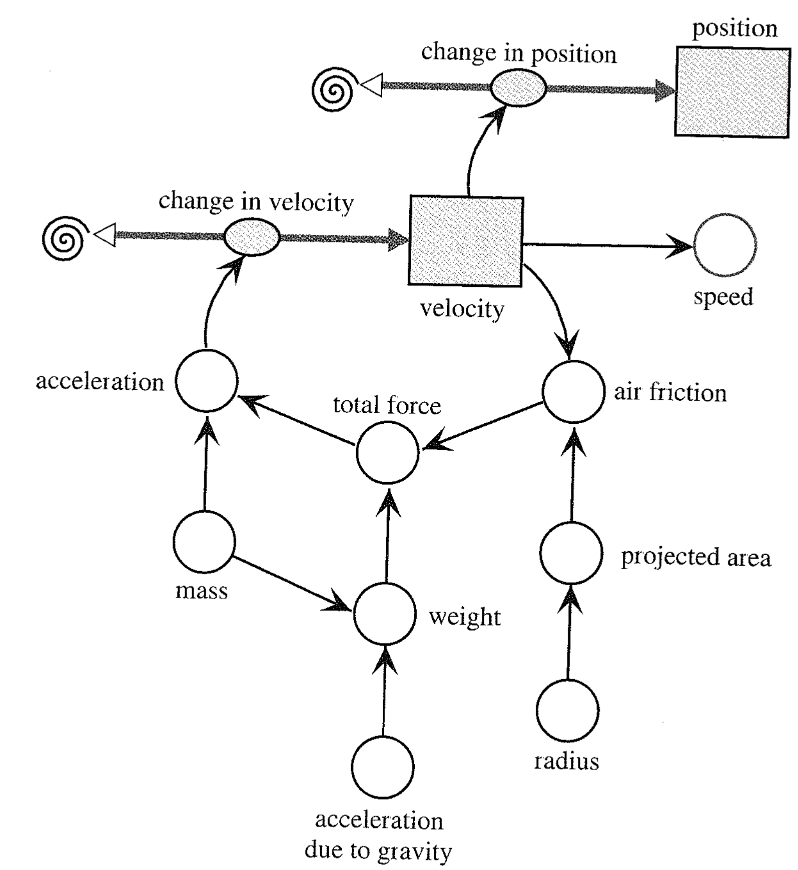

Figure 4.1.4 Diagram for motion of ball under influence of air friction; changes to converters/variables from Figure 4.1.1 in color

from a new *projected_area* converter to a new converter for *air_friction*. The *projected_area* converter/variable stores the cross-sectional, or projected, area of the object in the direction of motion. Assuming spherical objects, another converter/variable stores the *radius*; and the equation in *projected_area* is *pi* * *radius*^2, where *pi* is built in or an approximated constant 3.15169, depending on the system dynamics tool. Both forces, *weight* and *air_friction*, connect to a new converter/variable for *total_force*, which is the sum of the individual forces. Employing Newton's Second Law again with $a = F/m$, *acceleration* is *total_force/mass*. This acceleration provides the change in velocity for the flow into *velocity*.

Figure 4.1.4 contains a **feedback loop**. The initial value of air friction employs the initial velocity, here 0 m/sec; and *air_friction* contributes to the *total_force*, which *acceleration* uses. Acceleration is the *change_in_position*, which contributes to *velocity*. Then, the current value of *velocity* "feeds back" into *air_friction* for a new computation of that force.

To detect the influence of drag, we consider a ball of mass 0.5 kg and radius 0.05 m dropped (initial velocity = 0 m/sec) from a height of 400 m. Equation Set 4.1.1 presents various underlying equations for the model.

Equation Set 4.1.1

Various underlying equations to accompany diagram in Figure 4.1.4

mass = 0.5 kg
acceleration_due_to_gravity = –9.81 m/sec^2
radius = 0.05 m
weight = *mass* * *acceleration_due_to_gravity*
projected_area = 3.14159 * *radius*^2
air_friction = –0.65 * *projected_area* * *velocity* * *ABS*(*velocity*)
total_force = *weight* + *air_friction*
acceleration = *total_force/mass*
change_in_velocity = *acceleration*
change_in_position = *velocity*
speed = *ABS*(*velocity*)
velocity(0) = 0 m/sec
velocity(t) = *velocity*($t - \Delta t$) + (change_in_velocity) * Δt
position(0) = 400 m
position(t) = *position*($t - \Delta t$) + (*change_in_position*) * Δt

Running the simulation for 15 seconds, we see in Figure 4.1.5 that the ball reaches a constant, or **terminal speed**, of about 31 m/sec. From about time 6 seconds on, the position graph is almost linear, so that acceleration is approximately 0 m/sec^2.

Quick Review Question 6

At the terminal velocity, give the relationship between *weight* and *air_friction*: (A) *weight* < *air_friction*; (B) *weight* = *air_friction*; (C) *weight* > *air_friction*

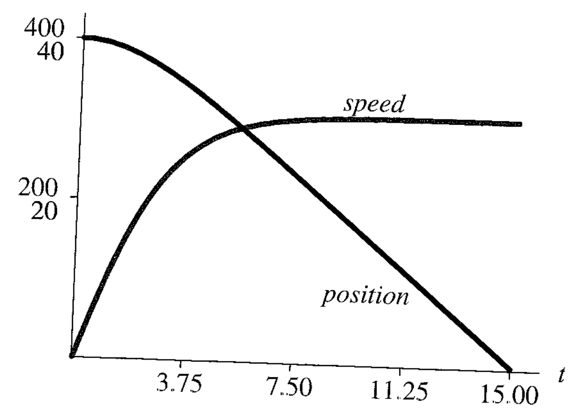

Figure 4.1.5 Graph of position (m) and speed (m/sec) of object under influence of friction

Quick Review Question 7

This question reflects on refinement of the model of Example 2 to incorporate skydiving. After completing this question and before continuing in the text, we suggest that you revise the model.

a. Give the phases of the fall during the simulation.
b. Give the variable whose value we can use to trigger the change in phase: *acceleration, mass, position, velocity, weight.*
c. Give the value(s) that change upon opening of the parachute: *acceleration_due_to_gravity, mass, projected_area, weight.*
d. Describe anticipated changes to the graphs in Figure 4.1.5 after deployment of a parachute.

Modeling a Skydive

Example 3

To model a skydive, we build heavily on Example 2 of a falling object. For simplicity, we consider someone jumping out of a stationary helicopter at 2000 m (about 6562 ft), and we ignore changes in air density. Project 5 considers parachuting out of a moving plane, which imparts a horizontal velocity to the jumper. The model for a skydive out of a helicopter has two phases, one where the person is in a free-fall and the other after the parachute opens when the larger surface area results in more air resistance. For our model, the main difference in these two phases is the projected area in the direction of motion, down. The cross-sectional area of a jumper in the stable arch position with arms arched back and legs bent at the knees is approximately 0.4 m^2 (about 4.3 ft^2). Parachutes vary in their designs, but 28 m^2 (about 301 ft^2) is a reasonable value. We trigger the pull of the ripcord by the height (*position*) above the ground, say, 1000 m (about 3281 ft). Thus, the diagram contains a converter/variable (*position_open*) for this quantity and

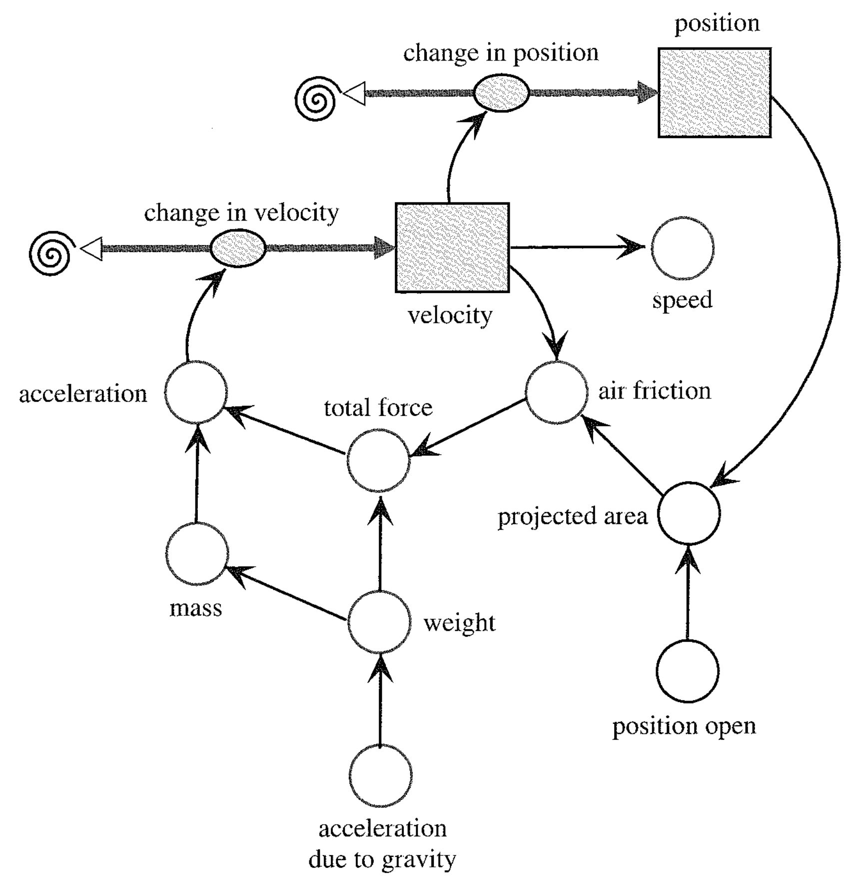

Figure 4.1.6 Diagram of skydiver's motion under influence of air friction

connectors/arrows from *position* to *projected_area* and from *position_open* to *projected_area*. Figure 4.1.6 presents a model diagram for this example with changes in color from Figure 4.1.4 on a ball's fall. Assuming the parachute fully opens instantaneously, the equation in *projected_area* is no longer a constant but employs the following logic:

if (*position* > *position_open*)
  projected_area ← 0.4
else
  projected_area ← 28

Figure 4.1.7 shows graphs of the position and speed of a 90-kg (comparable to about 198-lb) skydiver versus time. Until a height of 1000 m, which occurs at about 21.3 seconds into the fall from 2000 m, the skydiver is in a free-fall approaching a terminal speed of about 58.2 m/sec (about 130 mph). At 1000 m, the person pulls the ripcord, and in a very short amount of time, the parachutist's speed slows to a new terminal speed of 6.96 m/sec (about 15.6 mph).

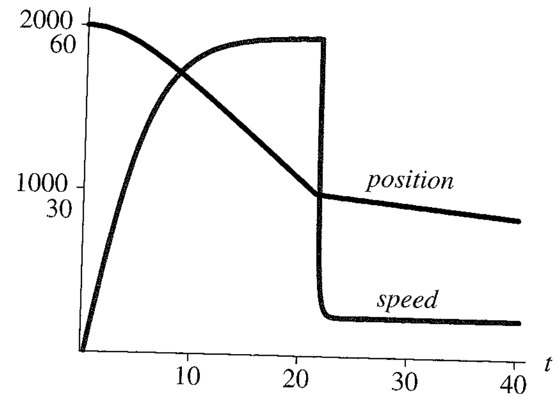

Figure 4.1.7 Position (m) and speed (m/sec) of skydiver

Quick Review Question 3

a. How does the terminal speed of a skydiver who curls into a ball compare to that of the same skydiver who is in a stable arch position?

A. Less **B.** Equal **C.** Greater

b. Referring to Figure 4.1.7, approximately how long does it take for the skydiver in free-fall to be close to terminal speed?

A. 13 sec **B.** 21 sec **C.** 40 sec

c. Referring to Figure 4.1.7, at approximately what time does the skydiver pull the ripcord?

A. 13 sec **B.** 21 sec **C.** 40 sec

Assessment of the Skydive Model

The shapes of the graphs of position and velocity in Figure 4.1.7 match the opening description of a skydive. However, our model exhibits a terminal speed of about 130 mph (about 58.2 m/sec), while actual, measured speeds of 110–120 mph are common. The drag coefficient of a jumper is probably larger than the assumed $C = 1$ of the model. Also, the example employs the sea-level density of air, while the air density at 10,000 ft (about 3,048 m) is about 73.8% (0.952 kg/m^3) of sea-level density. Adjusting the initial position to be 3048 m and using an air density of 0.952 kg/m^3 with the Newtonian friction of $F = 0.5CDAv^2$, the model indicates a terminal velocity of 68 m/sec (about 152 mph) for the free-fall for less than 35 seconds. However, the air density changes as the skydiver descends. Projects 4 and 7 explore refinements of the model to account for this variation. Project 5 also considers the skydiver jumping from a moving plane as opposed to a stationary helicopter.

Exercises

1. **a.** Using the equations and values of Example 1, write differential equations with initial conditions for acceleration and velocity.
   b. Using calculus or an appropriate computational tool, solve the differential equations of Part a to obtain velocity and position as functions of time.
2. Adjust *Fall* of Example 1 so that the object falls with an initial velocity of zero and initial position of 400 m. Compare the results with those in *FallFriction* of Example 2, which accounted for friction.
3. **a.** Using the equations and values of Example 2, write a differential equation involving the derivative of velocity for when an object reaches terminal velocity. At terminal velocity, the forces acting on the body are equal.
   b. Solve the equation of Part a using calculus or a computational tool.
4. Give the adjustments to the diagram in Figure 4.1.6 along with equations so that graphs of new converters/variables *adjusted_position* and *adjusted_speed* become horizontal lines at position 0 m after the parachutist lands.
5. Repeat Exercise 3 using Stokes' friction instead of Newtonian friction.
6. Suppose a raindrop evaporates as it falls but maintains its spherical shape. Assume that the rate at which the raindrop evaporates (that is, the rate at which it loses mass) is proportional to its surface area, where the constant of proportionality is –0.01. The density (mass per volume) of water at 3.98°C is 1 g/cm^3. The surface area of a sphere is $4\pi r^2$, and its volume is $4\pi r^3/3$, where r is the radius. Assume no air resistance. (Project 8 models the motion of this raindrop under the influence of air resistance.)
   a. Assume that the initial radius is 0.3 cm. Determine the raindrop's initial mass.
   b. Write a differential equation for the rate of change of mass with respect to time as a function of r.
   c. Write an equation for r as a function of mass.
7. Adjust Example 3 so that the parachute opening depends on time, not height above the ground.
8. Write a system of differential equations to represent Example 3.
9. Using the models in your system dynamics tool's *Fall* and *FallFriction* files (see "Download"), compare position graphs for a dropped object with and without consideration of friction. Also, consider the velocity graphs. Discuss the results.

Projects

For all model development, use an appropriate system dynamics tool.

1. Develop a model to estimate the total change in position of the car with velocities from Table 2.4.3 of Module 2.4, "Fundamental Concepts of Integral Calculus." Employ an input graph instead of an equation to record Table 2.4.3's values for the change in position. Give the absolute and relative errors of your estimate in comparison to the exact value of $203\frac{1}{3}$ m.

Table 4.1.2
Approximate Air Densities at Various Altitudes

Altitude (m)	*Density (kg/m³)*
0	1.290
610	1.216
1219	1.146
1829	1.078
2438	1.014
3048	0.952
3658	0.894
4267	0.839
4877	0.786

2. Using Stokes' friction, develop a model for the motion of a dust particle floating down from a height of 50 m. Using comparative plots, determine its terminal speeds for various values of Stokes' constant of proportionality.
3. A bathysphere is a pressurized metal vessel in the shape of a sphere that allows people to explore the ocean to much greater depths than are possible by skin diving. A ship lowers and raises the sphere using a steel cable and communicates with its two occupants by telephone. In the 1930s, explorers William Beebe and Otis Barton developed the first bathysphere, which weighed 4500 pounds and had a diameter of 4′9″. In a subsequent one, they descended to about 3000 ft in the ocean. Ignoring currents but not drag, model the sinking motion of a bathysphere. Assume that the boat reels out the steel cable fast enough so as not to affect the bathysphere's motion (Col 2000; Uscher 2000).
4. Table 4.1.2 contains air densities at various altitudes. Using these values on an input graph, refine the model for Example 3 (Aber and Aber 2003).
5. Suppose an airplane is traveling in a straight line horizontally at 130 m/sec at a height of 600 m when a parachutist jumps out of the plane at an angle of 30° with the horizon. Model the motion of the skydive.
6. Model the motion of a meteor falling to the earth. Assume an initial height of 100,000 m, initial velocity of −10,000 m/sec, coefficient of drag of 2, mass of 500 kg, and density of 8000 kg/m³ for iron or 3500 kg/m³ for stone (Schecker 1996). Give graphs for position, velocity, and acceleration versus time. Give comparison graphs for velocity versus height for meteors of various masses. Similarly, give comparison graphs for acceleration versus height. NASA's Glenn Research Center gives the following model for air density using variables D for density (slugs/ft³), P for pressure (lbs/ft²), T for temperature (°F), and h for altitude (ft):

$$D = \frac{P}{1718(T + 459.7)}, \quad \text{where}$$

for $h > 82{,}345$ ft, $T = -205.05 + 0.00164\,h$ and $P = 51.97\left(\dfrac{T + 459.7}{389.98}\right)^{-11.388}$

for $36{,}152 < h < 82{,}345$ ft, $T = -70$ and $P = 473.1e^{(1.73-0.000048h)}$; and

for $h < 36{,}152$ ft, $T = 59 - 0.00356h$ and $P = 2116\left(\dfrac{T + 459.7}{518.6}\right)^{5.256}$

Note, if you wish to use metric instead of English units, you can use the following: 1 slug = 14.5939 kg and 1 ft = 0.3048 m (Benson).

7. Using NASA's Glenn Research Center model for air density at heights less than 36,152 ft (see Project 6), refine the model in Example 3.
8. **a.** Model the change in mass of the raindrop that Exercise 5 describes.
   b. Model the motion of this raindrop taking into account air resistance.
9. Develop a model to compare the terminal velocities of objects of different masses, such as a mouse, cat, human, horse, elephant, etc. With the density of living protoplasm being almost constant across a wide variety of species, assume mass is proportional to the cube of a linear dimension, such as length or circumference; but surface area is proportional to the square of a linear dimension. How do the terminal velocities of more massive objects compare to those of less massive objects? Can a cat survive a fall from a tall building (Diamond 1989)?

Answers to Quick Review Questions

1. **a.** time, perhaps in seconds; distance, perhaps in meters; velocity, perhaps in m/sec; acceleration, perhaps in m/sec²
   b. $v(t) = \dfrac{ds}{dt}$
   c. $a(t) = \dfrac{dv}{dt}$
   d. Acceleration, which is acceleration due to gravity, −9.81 m/sec²
   e. s and v
   f. ds/dt
   g. dv/dt or a, which is the constant acceleration due to gravity without friction
2. **a.** *acceleration_due_to_gravity* = −9.81 m/sec²
   b. *change_in_velocity* = *acceleration_due_to_gravity*
   c. *change_in_position* = *velocity*
3. **a.** $m = F/a = 981\ \text{N}/(9.81\ \text{m/sec}^2) = 100\ \text{kg}$
   b. $a = F/m = 10\ \text{N}/(5\ \text{kg}) = 2\ \text{m/sec}^2$
4. **a.** $\dfrac{1\ \text{g}}{\text{cm}^3} \times \dfrac{1\ \text{kg}}{10^3\ \text{g}} \times \left(\dfrac{10^2\ \text{cm}}{1\ \text{m}}\right)^3 = 10^3\ \dfrac{\text{kg}}{\text{m}^3}$
   b. $F = -0.5CDAv|v| = -0.5(0.9)(10^3)(0.03)(-20)|-20| = 5400$ N
   c. `-0.65 * A * v * ABS(v)`
5. **a.** velocity and projected area
   b. $-0.65 * A * v * ABS(v)$
   c. weight and air friction

d. $m * g$
e. F/m

6. (B) *weight* = *air_friction*
7. a. Before and after opening of the parachute
   b. *position*
   c. *projected_area*
   d. The position curve should continue to decrease but not as steeply. The speed curve should suddenly drop and then level off to a new terminal velocity.
8. a. C. Greater because *projected_area* is less, causing *air_friction* to be less, making the absolute values of *total_force*, *acceleration*, *change_in_velocity*, *velocity*, and *speed* more.
   b. A. 13 sec
   c. B. 21 sec

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MODULE 4.2

Modeling Bungee Jumping

Downloads

The text's website has available for download with various system dynamics tools the following files containing the models in this module: *VerticalSpring* and *Bungee*.

Introduction

On April Fool's Day in 1979, four members of Oxford University's Dangerous Sports Club, dressed in tails and top hats, climbed out onto the Clifton Suspension Bridge in Bristol, U.K. Each attached one end of a nylon-braided, rubber shock cord to themselves and the other to the bridge. Then, they jumped off toward the 250-foot Avon Gorge. Voila! The sport of bungee jumping had begun in the western world.

What in the world possessed these men to do such a thing? The story goes that they watched a film on "land divers" from Pentecost Island in the South Pacific and became inspired to try diving themselves.

What are "land divers"? These divers are the male inhabitants of Pentecost who dive from platforms at various heights along a wooden tower. For these dives, lianas (vines) attached to the tower are tied to their ankles. Divers may be as young as seven years of age. Naturally, the lianas have to be selected very carefully. They must be just the right length and elasticity for the height of the platform and the weight of the diver. Consideration must be given to the length of the platform (which collapses and absorbs some shock), the slope of the land, and the swaying of the tower. A perfect dive will have the hair of the diver just brushing the ground. A miscalculation might be fatal. Land diving is part of ceremonies that ensure the yam harvest and fertility. Now, extreme-sports enthusiasts come from all over the world to experience "land diving."

How did this practice get started? Why would men choose to jump from platforms with vines tied to their ankles? The annual land dives are based on local lore